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Holographic dark energy in a cyclic universe

Jingfei Zhang[1,a], Xin Zhang[2,b], Hongya Liu[1,c]

[1] School of Physics and Optoelectronic Technology, Dalian University of Technology, Dalian 116024, P.R. China
[2] Kavli Institute for Theoretical Physics China, Institute of Theoretical Physics, Chinese Academy of Sciences (KITPC/ITP-CAS), P.O. Box 2735, Beijing 100080, P.R. China

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Abstract. In this paper we study the cosmological evolution of the holographic dark energy in a cyclic universe, generalizing the model of holographic dark energy proposed by Li. The holographic dark energy with $c<1$ can realize a quintom behavior; namely, it evolves from a quintessence-like component to a phantom-like one. The holographic phantom energy density grows rapidly and dominates the late-time expanding phase, helping to realize a cyclic universe scenario in which the high energy regime is modified by the effects of quantum gravity, causing a turn-around (and a bounce) of the universe. The dynamical evolution of holographic dark energy in the regimes of low energy and high energy is governed by two differential equations, respectively. It is of importance to link the two regimes for this scenario. We propose a link condition giving rise to a complete picture of holographic evolution of a cyclic universe.

The astronomical observations over the past decade imply that our universe is currently dominated by dark energy, which leads to an accelerated expansion of the universe (see e.g. [1–7]). The combined analysis of cosmological observations suggests that the universe is spatially flat and consists of about 70% dark energy, 30% dust matter (cold dark matter plus baryons), and negligible radiation. The basic characteristic of dark energy is that its equation of state parameter w (the definition of w is $w=p/\rho$, where ρ is the energy density and p is the pressure) has a negative value ($w<-1/3$). The most obvious candidate for dark energy is the cosmological constant λ [8] for which $w=-1$ (for reviews see e.g. [9–14]). However, the cosmological constant always suffers from the "fine-tuning" and "cosmic coincidence" problems. Another candidate for dark energy is the energy density associated with a dynamical scalar field; a slowly varying, spatially homogeneous component. Typical examples of such a type of dark energy are the so-called "quintessence" [15–24] and "phantom" [25, 26] models. Quintessence dark energy provided by a canonical scalar field has an equation of state $w>-1$, while the phantom energy associated with a scalar field with a negative-kinetic energy has an equation of state $w<-1$. It is remarkable that for phantom dark energy in this scenario all the energy conditions in general relativity (including the weak energy condition) are violated. Due to such a supernegative equation of state, the phantom component leads to a "big rip" singularity at a finite future time, at which all bound objects will be torn apart.

In the phantom scenario, generically, there exist two space-time singularities in the universe; one is the initial "big bang" singularity, the other is the future "big rip" singularity. The space-time singularities are disgusting for the majority of theorists; thus a mechanism for avoiding the initial and future singularities are attractive for physicists and cosmologists. An effective way for eliminating the singularities is to introduce a ρ^2 term with a negative sign to the Friedmann equation if the energy is very high. Such a modified Friedmann equation with a phantom energy component leads to a cyclic universe scenario in which the universe oscillates through a series of expansions and contractions. Phantom energy can dominate the universe today and drive the current cosmic acceleration. Then, as the universe expands, it becomes more and more dominant and its energy density becomes very high. When the phantom energy density reaches a critical value, a very high energy density, the universe reaches a state of maximum expansion, which we call the "turn-around" point, and then it begins to recollapse, according to the modified Friedmann equation. The contraction of the universe makes the phantom energy density dilute away and the matter density dominate. Once the universe reaches its smallest extent, the matter density hits the value of the critical density, the modified Friedmann equation leads to a "bounce", making the universe once again begin to expand.

The idea of an oscillating universe was first proposed by Tolman in the 1930's [27, 28]. In recent years, Steinhardt, Turok and collaborators [29–33] proposed a cyclic model of the universe as an alternative to the inflation scenario, in

[a] e-mail: jfzhang@student.dlut.edu.cn
[b] e-mail: zhangxin@itp.ac.cn
[c] e-mail: hyliu@dlut.edu.cn

which the cyclicity of the universe is realized in the light of two separated branes. The cyclic scenario discussed in this paper is distinguished from the Steinhardt–Turok cyclic scenarios, in that the phantom energy plays a crucial role. In the oscillating or cyclic models, the principal obstacles against success come from the problems of black holes and entropy. For the discussions of the problems of black holes and entropy in the cyclic scenario, see e.g. [34–36].

Usually, the phantom energy density becomes infinite in a finite time, leading to the big rip singularity. However, we expect that an epoch of quantum gravity sets in before the energy density reaches infinity. Therefore, we arrive at the notion that quantum gravity governs the behavior of the universe both at the beginning and at the end of the expanding universe, where the energy density is enormously high. This high energy density physics may lead to modifications to the Friedmann equation, such as in loop quantum cosmology [37–40] and braneworld scenarios [41–43], which causes the universe to bounce when it is small, and to turn around when it is large. At high energy densities, we employ the modified Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho\left(1-\frac{\rho}{\rho_{\rm c}}\right), \qquad (1)$$

where $H=\dot{a}/a$ is the Hubble parameter, G is the Newton gravity constant, and $\rho_{\rm c}$ is the critical energy density set by quantum gravity, distinguished from the usual critical density $3M_{\rm pl}^2H^2$ (where $M_{\rm pl}=1/\sqrt{8\pi G}$ is the reduced Planck mass). This modified Friedmann equation can be derived from the effective theory of loop quantum cosmology [37–40], and also from the braneworld scenario [43]. In loop quantum cosmology, the critical energy density can be evaluated as $\rho_{\rm c}\approx 0.82\rho_{\rm pl}$, where $\rho_{\rm pl}=G^{-2}=2.22\times10^{76}\,{\rm GeV}^4$ is the Planck density. In the braneworld scenario, $\rho_{\rm c}=2\sigma$, where σ is the brane tension, and a negative sign in (1) can arise from a second time-like dimension; but that gives difficulties with closed time-like paths. In models motivated by the Randall–Sundrum scenario [41, 42], the most natural energy scale of the brane tension is of the order of the Planck mass, but the problem can generally be treated for any value of $\sigma > {\rm TeV}^4$. Once the energy density of the universe reaches the critical density $\rho_{\rm c}$, the universe changes its evolution direction. At that energy scale, if it has been expanding, it turns around and begins to contract; if it has been contracting, it bounces and begins to expand. Modifications to the Friedmann equation thus motivate the bounce and the turn-around, both of which are nonsingular.

Recently, considerable interest has been stimulated in explaining the observed dark energy by the holographic dark energy model. The holographic dark energy model is an attempt to probe the nature of dark energy within the framework of a fundamental theory originating from some considerations of the features of quantum gravity theory. Cx oncretely speaking, this model is constructed in the light of the holographic principle [44, 45] of quantum gravity. In the holographic scenario, the dark energy is a dynamically evolving vacuum energy density that can realize the phantom behavior. If the holographic dark energy becomes a phantom, this scenario will involve a big rip singularity in the far future unless the Friedmann equation gets a quantum gravity correction in the high energy regime as shown in (1). Thus, phantom-like holographic dark energy can play a crucial role in realizing a cyclic universe scenario, and inversely, a cyclic universe can endow the holographic dark energy with peculiar features. In this paper, we shall study the cyclic universe with a holographic phantom and investigate the characteristic of the holographic dark energy in such a cyclic universe.

According to the holographic principle, the number of degrees of freedom for a system within a finite region should be finite and should be bounded roughly by the area of its boundary. In the cosmological context, the holographic principle will set an upper bound on the entropy of the universe. Motivated by the Bekenstein entropy bound, it seems plausible that one may require that for an effective quantum field theory in a box of size L with UV cutoff Λ, the total entropy should satisfy $S=L^3\Lambda^3\le S_{\rm BH}\equiv \pi M_{\rm pl}^2L^2$, where $S_{\rm BH}$ is the entropy of a black hole with the same size L. However, Cohen et al. [46] pointed out that to saturate this inequality some states with Schwartzschild radius much larger than the box size have to be counted in. As a result, a more restrictive bound, the energy bound, has been proposed to constrain the degrees of freedom of the system, requiring the total energy of a system with size L not to exceed the mass of a black hole with the same size, namely, $L^3\Lambda^4=L^3\rho_\Lambda\le LM_{\rm pl}^2$. This means that the maximum entropy is in the order of $S_{\rm BH}^{3/4}$. When we take the whole universe into account, the vacuum energy related to this holographic principle is viewed as dark energy, usually dubbed holographic dark energy. The largest IR cut-off L is chosen by saturating the inequality, so that we get the holographic dark energy density

$$\rho_\Lambda = 3c^2M_{\rm pl}^2L^{-2}, \qquad (2)$$

where c is a numerical constant (note that $c>0$ is assumed), and as usual $M_{\rm pl}$ is the reduced Planck mass. Hereafter, we will use the unit $M_{\rm pl}=1$ for convenience. It has been conjectured by Li [47] that the IR cutoff L should be given by the future event horizon of the universe,

$$R_{\rm eh}(a) = a\int_t^\infty \frac{{\rm d}t'}{a(t')} = a\int_a^\infty \frac{{\rm d}a'}{Ha'^2}. \qquad (3)$$

Such a holographic dark energy looks reasonable, since it may provide simultaneously natural solutions to both dark energy problems, as demonstrated in [47]. The holographic dark energy model has been tested and constrained by various astronomical observations [48–57]. For other extensive studies on the holographic dark energy, see e.g. [58–75].

The holographic dark energy scenario reveals the dynamical nature of the vacuum energy. When taking the holographic principle into account, the vacuum energy density will evolve dynamically. The dimensionless parameter c plays a crucial role in the holographic evolution of the universe. As has been pointed out in [48–50], the value of c determines the destiny of the holographic universe. When

$c \geq 1$, the equation of state of dark energy will evolve in the region of $-1 \leq w \leq -1/3$. In particular, if $c = 1$ is chosen, the behavior of the holographic dark energy will be more and more like a cosmological constant with the expansion of the universe, such that ultimately the universe will enter the de Sitter phase in the far future. When $c < 1$, the holographic dark energy will exhibit a quintom-like evolution behavior (for "quintom" dark energy; see, e.g., [76–84] and references therein), i.e., the holographic evolution will make the equation of state cross $w = -1$ (from $w > -1$ it evolves to $w < -1$). That is to say, $c < 1$ makes the holographic dark energy today behave as a phantom energy that leads to a cosmic doomsday ("big rip") in the future. Nevertheless, as discussed above, at high energy densities the Friedmann equation may be modified to (1) due to some possible quantum gravity effects, which can successfully eliminate the big rip singularity. It is remarkable that the analyses of the observational data imply that the value of c in the model of holographic dark energy is very likely less than 1 [48–50], i.e., the holographic dark energy is very possibly behaving as a phantom energy presently. Intriguingly, then, considering the modified Friedmann equation (1) in the high energy regime, the holographic dark energy (with $c < 1$) along with some dust-like matter components can realize a cyclic universe scenario in which the cosmological evolution is nonsingular.[1]

First, consider the low energy regime of the universe, $\rho \ll \rho_c$. At this regime, for the universe we have the usual Friedmann–Robertson–Walker (FRW) case, $3H^2 = \rho$. Consider a universe filled with a matter component ρ_m (including both baryon matter and cold dark matter) and a holographic dark energy component ρ_Λ; then the Friedmann equation reads

$$3H^2 = \rho_m + \rho_\Lambda \,. \tag{4}$$

Defining the fractional densities $\Omega_\Lambda = \rho_\Lambda/3H^2$ and $\Omega_m = \rho_m/3H^2 = \Omega_m^0 H_0^2 H^{-2} a^{-3}$, where a is the scale factor of the universe and $a_0 = 1$ has been set, the Friedmann equation (4) can also be rewritten as

$$\frac{H^2}{H_0^2} = \Omega_m^0 a^{-3} + \Omega_\Lambda \frac{H^2}{H_0^2} \,. \tag{5}$$

Combining the definition of the holographic dark energy (2) and the definition of the future event horizon (3), we derive

$$\int_a^\infty \frac{\mathrm{d}\ln a'}{Ha'} = \frac{c}{Ha\sqrt{\Omega_\Lambda}} \,. \tag{6}$$

[1] In general, cyclic universe models confront two severe problems making the infinite cyclicity impossible. First, the black holes in the universe, which cannot disappear due to the Hawking area theorems, grow ever larger during subsequent cycles, and they eventually will occupy the entire horizon volume during the contracting phase, so that the calculations break down. The second problem is that the entropy of the universe increases from cycle to cycle due to the second law of thermodynamics, so that extrapolation into the past will lead back to an initial singularity. In this paper, we do not consider these difficulties.

We notice that the Friedmann equation (5) implies

$$\frac{1}{Ha} = \sqrt{a(1-\Omega_\Lambda)} \frac{1}{H_0\sqrt{\Omega_m^0}} \,. \tag{7}$$

Substituting (7) into (6), one obtains the following equation:

$$\int_x^\infty \mathrm{e}^{x'/2}\sqrt{1-\Omega_\Lambda}\,\mathrm{d}x' = c\mathrm{e}^{x/2}\sqrt{\frac{1}{\Omega_\Lambda} - 1} \,, \tag{8}$$

where $x = \ln a$. Then taking the derivative with respect to x in both sides of the above relation, we easily get the dynamics satisfied by the dark energy, namely the differential equation for the fractional density of the dark energy,

$$\Omega'_\Lambda = \Omega_\Lambda(1-\Omega_\Lambda)\left(1 + \frac{2}{c}\sqrt{\Omega_\Lambda}\right) , \tag{9}$$

where the prime denotes the derivative with respect to $x = \ln a$. This equation describes the behavior of the holographic dark energy in the low energy regime completely, and it can be solved exactly [47]:

$$\begin{aligned}\ln\Omega_\Lambda &- \frac{c}{2+c}\ln\left(1-\sqrt{\Omega_\Lambda}\right) + \frac{c}{2-c}\ln\left(1+\sqrt{\Omega_\Lambda}\right) \\ &- \frac{8}{4-c^2}\ln\left(c+2\sqrt{\Omega_\Lambda}\right) = \ln a + y_0 \,, \end{aligned} \tag{10}$$

where y_0 is an integration constant that can be determined by setting today as an initial condition,

$$\begin{aligned} y_0 = {} & \ln\left(1-\Omega_m^0\right) - \frac{c}{2+c}\ln\left(1-\sqrt{1-\Omega_m^0}\right) \\ & + \frac{c}{2-c}\ln\left(1+\sqrt{1-\Omega_m^0}\right) \\ & - \frac{8}{4-c^2}\ln\left(c+2\sqrt{1-\Omega_m^0}\right) . \end{aligned} \tag{11}$$

From the energy conservation equation of the dark energy, the equation of state of the dark energy can be given [47]:

$$w = -1 - \frac{1}{3}\frac{\mathrm{d}\ln\rho_\Lambda}{\mathrm{d}\ln a} = -\frac{1}{3}\left(1 + \frac{2}{c}\sqrt{\Omega_\Lambda}\right) . \tag{12}$$

Note that the formula $\rho_\Lambda = [\Omega_\Lambda/(1-\Omega_\Lambda)]\rho_m^0 a^{-3}$ and the differential equation of Ω_Λ, see (9), are used in the second equality sign.

As time passes by, the dark energy gradually dominates the evolution of the universe, Ω_Λ increases to 1, and the most important term on the left-hand side of (10) is the second term; thus for large a, we get

$$\sqrt{\Omega_\Lambda} = 1 - 2^{\frac{2+c}{2-c}}(2+c)^{\frac{8}{c(c-2)}}\mathrm{e}^{-\frac{2+c}{c}y_0}a^{-\frac{2+c}{c}} \,. \tag{13}$$

Since the universe is dominated by the dark energy for large a, we have

$$\rho_\Lambda \simeq 3H^2 = \frac{\rho_m}{1-\Omega_\Lambda} = \frac{\rho_m^0 a^{-3}}{1-\Omega_\Lambda} \,. \tag{14}$$

Thus, using (13) in the above relation, we derive

$$\rho_\Lambda = 2^{-\frac{2(2+c)}{2-c}}(2+c)^{-\frac{8}{c(c-2)}}\mathrm{e}^{\frac{2+c}{c}y_0}\rho_\mathrm{m}^0 a^{\frac{2(1-c)}{c}}\,. \tag{15}$$

It can be clearly seen from the above expression that the value of c plays a significant role in determining the final evolution of the dark energy. When $c=1$, the holographic dark energy will become a cosmological constant that is related to ρ_m^0 through the above relation without a. The choice of $c>1$ makes the density of the dark energy continuously decrease, just like the case of quintessence dark energy. When $c<1$, the density of dark energy ceaselessly increases with the expansion of the universe. The phantom behavior ($c<1$) results in the density of holographic dark energy becoming enormously high when a is very large. As the universe goes into the high energy regime, quantum gravity begins to operate, giving rise to the modified Friedmann equation, (1). However, it is very hard to justify the borderline between the usual Friedmann equation and the modified Friedmann equation. Namely, one cannot accurately say when the usual Friedmann equation is replaced by the modified one. Hence, we have to set a criterion by hand. One can assume that quantum gravity begins to operate when $\rho_\Lambda = \eta\rho_\mathrm{c}$, where, say, $\eta \sim \mathcal{O}(10^{-3}) - \mathcal{O}(10^{-2})$. We thus derive, at that moment, the scale factor of the universe,

$$a_\eta = \left[\eta 2^{\frac{2(2+c)}{2-c}}(2+c)^{\frac{8}{c(c-2)}}\mathrm{e}^{-\frac{2+c}{c}y_0}\left(\frac{\rho_\mathrm{c}}{\rho_\mathrm{m}^0}\right)\right]^{\frac{c}{2(1-c)}}. \tag{16}$$

We assume that when $a>a_\eta$ the evolution of the universe is governed by (1). Here, any other forms of energy have already been decayed away, and the dark energy is thus the unique component of the universe; so we have

$$3H^2 = \rho_\Lambda\left(1-\frac{\rho_\Lambda}{\rho_\mathrm{c}}\right). \tag{17}$$

From this relation, we have

$$\Omega_\Lambda = 1+\frac{\rho_\Lambda}{\rho_\mathrm{c}-\rho_\Lambda}\,. \tag{18}$$

This indicates that $\Omega_\Lambda>1$ when $a>a_\eta$, even though the space of the universe is assumed to be flat. In concrete terms, when $\rho_\Lambda \ll \rho_\mathrm{c}$, we have $\Omega_\Lambda \to 1^+$; when $\rho_\Lambda \to \rho_\mathrm{c}$, we have $\Omega_\Lambda \to \infty$. And, as a contrast, see (13), for large a but $a<a_\eta$, we have $\Omega_\Lambda \to 1^-$. Now the question naturally arises of how to realize the $\Omega_\Lambda = 1$ crossing. This question is actually equivalent to the one of asking when the usual Friedmann equation should be replaced by the modified one. The transition of the two phases is ambiguous so that we have to set a connection when $\rho_\Lambda = \eta\rho_\mathrm{c}$. Hence, the initial stage for (17) is from $\Omega_\Lambda = 1+\epsilon$, where ϵ is a small positive number.

The modified Friedmann equation can be rewritten as

$$\tilde{h}^2 = \frac{H^2}{\rho_\mathrm{c}} = \frac{\Omega_\Lambda - 1}{3\Omega_\Lambda^2}\,, \tag{19}$$

where the dimensionless parameter $\tilde{h}$ is positive for an expanding universe and negative for a contracting universe. Here, since an expanding universe is considered for illustration, we take the positive value to $\tilde{h}$. Combining the definitions of holographic dark energy and future event horizon, namely (2) and (3), yields

$$\int_a^\infty \frac{\mathrm{d}\ln a'}{\tilde{h}a'} = \frac{c}{\tilde{h}a\sqrt{\Omega_\Lambda}}\,. \tag{20}$$

Following (19), we have

$$\frac{1}{\tilde{h}a} = \frac{\sqrt{3}\Omega_\Lambda}{a\sqrt{\Omega_\Lambda - 1}}\,. \tag{21}$$

Substituting (21) into (20) yields

$$\int_x^\infty \mathrm{d}x' \frac{\Omega_\Lambda}{\mathrm{e}^{x'}\sqrt{\Omega_\Lambda - 1}} = \frac{c\sqrt{\Omega_\Lambda}}{\mathrm{e}^{x}\sqrt{\Omega_\Lambda - 1}}\,, \tag{22}$$

where $x=\ln a$. Then, taking the derivative with respect to x in both sides of this equation, one obtains a differential equation for the fractional density of the holographic dark energy,

$$\Omega_\Lambda' = 2\Omega_\Lambda(\Omega_\Lambda - 1)\left(\frac{1}{c}\sqrt{\Omega_\Lambda} - 1\right), \tag{23}$$

where the prime denotes the derivative with respect to $\ln a$. This differential equation governs the holographic evolution of the universe for the high energy regime. Note that here $c<1$, and $\Omega_\Lambda>1$; hence Ω_Λ' is always positive, namely the fractional density of dark energy increases in time, the correct behavior as we expect. This equation can be solved exactly, and the solution is

$$\ln\sqrt{\Omega_\Lambda} + \frac{c}{2(1-c)}\ln\left(\sqrt{\Omega_\Lambda}-1\right) - \frac{c}{2(1+c)}\ln\left(\sqrt{\Omega_\Lambda}+1\right) - \frac{1}{1-c^2}\ln\left(\sqrt{\Omega_\Lambda}-c\right) = \ln a + y_\eta\,, \tag{24}$$

where y_η is an integration constant that can be determined by an appropriate initial condition. Now let us deduce the equation of state for the holographic dark energy in the high energy regime ($a>a_\eta$). Following the energy conservation equation of dark energy, we have $w=-1-(1/3)(\mathrm{d}\ln\rho_\Lambda/\mathrm{d}\ln a)$. Writing

$$\rho_\Lambda = 3\Omega_\Lambda\tilde{h}^2\rho_\mathrm{c} = \frac{\Omega_\Lambda - 1}{\Omega_\Lambda}\rho_\mathrm{c}\,, \tag{25}$$

one can easily obtain

$$w = -\frac{1}{3}\left(1+\frac{2}{c}\sqrt{\Omega_\Lambda}\right). \tag{26}$$

Interestingly, this relation is the same as in the low energy regime; see (12). Note that here Ω_Λ is governed by (23).

It has been pointed out that the borderline between the usual Friedmann equation and the modified one is rather ambiguous. One has to designate a contrived link condition; for example, one can assume that the transition has occurred when $\rho_\Lambda = \eta\rho_\mathrm{c}$, where $\eta \sim \mathcal{O}(10^{-3}) - \mathcal{O}(10^{-2})$, say. Therefore, the initial stage of the high energy regime is contrived to be from $a = a_\eta$, where $\Omega_\Lambda = 1+\epsilon$ with $\epsilon = \eta/(1-\eta)$. At the moment of $a = a_\eta$ and $\Omega_\Lambda \to 1^+$, the most important term on the left-hand side of (24) is the second term; we thus can determine the integration constant:

$$y_\eta = \frac{c}{2(1-c)} \ln \frac{\eta}{2(1-\eta)} - \frac{c}{2(1+c)} \ln 2 - \frac{1}{1-c^2} \ln(1-c) - \ln a_\eta \,, \tag{27}$$

where a_η is given by (16). It is remarkable that for a large Ω_Λ, when the universe approaches the turn-around point, the left-hand side of (24) goes to zero. This gives the scale factor corresponding to the turn-around,

$$a_{\max} = \mathrm{e}^{-y_\eta} \,, \tag{28}$$

where y_η is given by (27). Namely, the maximum scale factor (at the turn-around) of the universe in the cyclic universe scenario with holographic dark energy is totally determined by the constant y_η. It should be noted that the link condition $\rho_\Lambda = \eta\rho_\mathrm{c}$ is rather important in this scenario, even though it is somewhat artificial. The reason for contriving such a link condition is that the borderline between the classical and quantum gravities is not so clear. Next, let us give several numerical examples. We take $c = 0.8$, $\Omega_\mathrm{m}^0 = 0.27$, and $h = 0.72$ (here h is the dimensionless Hubble parameter of today), which gives rise to $y_0 = -1.54$. The critical density of the universe depends on the theory we use; if we use an effective theory of loop quantum cosmology, we have $\rho_\mathrm{c} \approx 0.82\rho_\mathrm{pl} = 1.82\times10^{76}\,\mathrm{GeV}^4$; if we use a braneworld scenario, we can treat the value of ρ_c from TeV^4 to $10^{76}\,\mathrm{GeV}^4$. Here we take the loop quantum cosmology for illustration. The present density of dust matter is $\rho_\mathrm{m}^0 = 1.13\times10^{-47}\,\mathrm{GeV}^4$. Then we can determine the values of a_η and $a_{\max}$ if the value of η is given. We only show two examples, $\eta = 10^{-3}$ and $\eta = 10^{-2}$. The choice of $\eta = 10^{-3}$ gives $a_\eta = 2.86\times10^{240}$ and $a_{\max} = 1.53\times10^{245}$; the choice of $\eta = 10^{-2}$ results in $a_\eta = 2.86\times10^{242}$ and $a_{\max} = 1.50\times10^{245}$.

In fact, this scenario has a fatal flaw, namely, a cyclic universe has no future event horizon in principle, since an observer can eventually see the whole universe due to the cyclicity of the universe, if he/she waits a sufficiently long time. Therefore, the calculations in this paper break down in this regard. However, we can rescue the model by reconsidering the IR cutoff of the universe. In the original work of the holographic dark energy [47], the choice of the future event horizon as an IR cutoff is only a conjecture for ensuring the acceleration of the universe. Now that the future event horizon cannot exist in a cyclic universe, we might as well make a modification to the future event horizon. We can define a "finite-future event horizon", by replacing the infinity with a time T in the upper limit of integration in (3), where T denotes the time of the turn-around. Choosing the finite-future event horizon as the IR cutoff of the universe undoubtedly makes the holographic dark energy meaningful in the cyclic universe, at least in the expanding stage. However, for the contracting stage of the cyclic universe, the IR cutoff is ambiguous for us. We can steer clear of this difficulty by considering the contracting stage as a time-reversal course of the expanding stage. It should be admitted that this assumption is rather strong. It should also be noted that the emphasis of this paper is placed on the holographic evolution in the high energy regime in the expanding branch and on how to link the low and high energy regimes.

When the holographic phantom density reaches the critical density ρ_c, the universe starts to turn around and contract. In the contraction phase, the physical rules are assumed to be totally the same as in the expansion phase, i.e., the high energy regime is governed by (17) and the low energy regime is dictated by (4). The only difference is that the Hubble parameter has a negative value, but this does not affect the evolutionary rules of the holographic dark energy; i.e., the dynamical evolution of the holographic dark energy is still controlled by the differential equations (9) and (23). As the universe contracts, at first the energy density of the universe decreases, because the holographic phantom density decreases in importance (then the quintom behavior makes the holographic dark energy become a quintessence-like component whose density increases as the universe contracts, although not as strongly as matter and radiation do), but then it again increases as matter and radiation become dominant. Eventually, the energy density reaches the high values at which the modifications to the Friedmann equation become important. Once the energy density again hits the same critical density ρ_c, the universe stops contracting, bounces, and once again expands. The bounce looks like a "big bang" for us, and at this point the universe has its smallest extent (smallest scale factor $a_{\min}$) and largest energy density (the critical density ρ_c). An inflationary period may occur if the inflaton field can be excited by some mechanism that can solve the flatness and horizon problems, etc., and this can also generate the scale-invariant primordial perturbations seeding structure formation. For the inflationary universe in loop quantum cosmology, see, e.g., [85–88]. As the universe expands, its density deceases, and it goes through the radiation dominated and matter dominated periods, with the usual primordial nucleosynthesis, microwave background, and large structure formation. Around a redshift $z \sim \mathcal{O}(1)$, the universe begins to accelerate due to the existence of dark energy (the holographic dark energy in this paper). The holographic dark energy with $c < 1$ can help realize the turn-around discussed above. It is noteworthy that the cyclic universe discussed in this paper is an ideal case, and there are still several severe obstacles existing in cyclic cosmology, such as density fluctuation growth in the contraction phase, black hole formation, and entropy increase, which can obstruct the realization of a truly cyclic cosmology. These problems are not addressed in this paper.

To summarize, in this paper we investigated the holographic dark energy in a cyclic universe. We generalized the model of holographic dark energy proposed in [47] to the case of a cyclic universe, and we studied the cosmological evolution of the holographic dark energy in such a universe in detail. The holographic dark energy with $c < 1$ can realize a quintom behavior; namely, it evolves from a quintessence-like component to a phantom-like component. The phantom energy density will become very large in the far future, which leads to a "big rip" singularity at a finite time in which all bounded objects are finally disrupted. However, when the energy scale becomes enormously large, quantum gravity effects may bring significant modifications to the Friedmann equation, leading to the possibility of the avoidance of singularities. A modified Friedmann equation, (1), along with a phantom component and other matter components can realize a cyclic universe scenario in which the cosmic evolution is nonsingular. In such a cyclic scenario, the large densities cause the universe to bounce when it is small, and to turn around when it is large. We investigated the holographic phantom cosmological evolution ($c < 1$) in such a cyclic universe in detail. The dynamical evolution of holographic dark energy in the low energy regime and in the high energy regime are rather different; they are governed by two differential equations, respectively. Linking the two regimes together is a very important mission for this scenario. We proposed a link condition connecting the regimes of low energy and high energy, which gives rise to a complete picture of the holographic evolution of the cyclic universe.

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